ASPECTS OF TOPOLOGIC STRUCTURAL SYNTHESIS
OF THE PARALLELL MANIPULATORS

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Abstract: In the present paper a new formula is deduced for the calculus of mobility or degree of freedom (DOF) of complex mechanisms and manipulators by pointing out the rank of each independent closed contour (loop). Parallel manipulators are complex topological structures, having the kinematical pairs (joints) of various types. The kinematical chain is defined as a parallel structure between two kinematical chains (platforms). By the use of the new DOF formula a method of structural-topological synthesis for the kinematical schema for parallel manipulators.

1. Introduction

Parallel manipulators are multi – mobile planar or spatial mechanisms with two to six kinematical chains which connect two rigid bodies, generically named platforms.

Kinematical chains mounted in parallel between two platforms are composed of two or more kinematical elements, which are realized as open chains or as complex chains with closed and open contours [1].

Gough’s machine (1947) and the simulator type Stewart’s platform (1965) are constituted as spatial mechanisms with six identical kinematical chains [4]; these mobile platforms represented the initial structural model, from which subsequent research was started, using the name of parallel manipulator (PMp). The PMp-s can be connected in series, obtaining overlapping PMp-s, similar to the topological structure of the serial elephant trunk type robots.

It is observed that unlike the serial manipulators, with PMp-s practically all types of kinematical pairs are used [1], which allows a larger diversity of kinematical schemas as compared to serial manipulators - robots.

The parallel manipulator - robot is a mobile mechanic structure, with one or more levels, having closed kinematical chains, which are driven by actuators, with interesting particular features [4].

In the last period, since 1990, the attention of more researchers [5-9] has been oriented to approaching the potential possibilities of parallel manipulators. The results of numerous studies [4] show that the concept of the parallel manipulator has been generalized and can be used as a model of geometrical and kinematical analysis and synthesis in diverse domains, from the simulation platforms (in the aeronautic and automotive industries) and drilling petroleum platforms to industrial robots and leg mobile robots.

J. P. Merlet [4] defined a PMp as comprising a terminal organ with $n$ degrees of freedom and a fixed base, linked together by at least two independent kinematical chains, which are actuated by $n$ simple actuators.

2. General Formula for the Mobility of Complex Structure Manipulators

It is noted with $r$ the rank of the space associated to an independent loop and with $N$ the number of independent closed contours (of the same rank: $2 \leq r \leq 6$). The number of loops ($N_r$) is function of the number $n$ of links and the number ($C_m$) of all joints of various classes having the function class $m$:

$$N_r = \sum_{m=1}^{r} C_m - n + 1 \quad (2.1)$$

In the articulated manipulators (having $r=3$) the maximal function class of the joints is $m=2$, such as with planar manipulators. The minimal function class ($m=1$) is obtained for $r=2$, like in the case of manipulators with screw.

It is known [1, 2 and 3] that the formula of mobility-degree of freedom (DOF) - for the three dimensional manipulators, having one or more independent closed contours of the same rank $r$ is of the form:

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\[ M_r = r \cdot (n - 1) - \sum_{m=1}^{r-1} (r - m) \cdot C_m \]  

The formula (2) can be written as:

\[ M_r = \sum_{m=1}^{r-1} (m \cdot C_m) - r \cdot (\sum_{m=1}^{r-1} C_m - n + 1) \]  

or, observing formula (1) is deduced:

\[ M_r = \sum_{m=1}^{r-1} (m \cdot C_m) - r \cdot N_r \]  

For a complex spatial manipulator with two or more contours of various ranks \((2 \leq r \leq 6)\), formula (4) is generalized, obtaining a new general formula [1, 2]:

\[ M = \sum_{m=1}^{r-1} (m \cdot C_m) - \sum_{r=1}^{6} (r \cdot N_r) \]  

In formula (5) it was noted with \(N_r\) the number of closed contours (of the \(r\) rank), which is calculated function of \(C_m\), the number of the joints of functional class \(m \in [1,(r-1)]\), and of the number of links \((n)\) from the topological structure of the \(r\) rank closed contour, according to the actualized formula (1):

\[ N_r = \sum_{m=1}^{r} C_m - n_r + 1 \]  

In the particular case of manipulators having only open contours, the number of closed contours is \(N_r=0\), so formula (5) becomes:

\[ M = \sum_{m=1}^{r-1} (m \cdot C_m) \]  

3. Synthesis of Kinematical Chains with Actuators for PMp-s

Unlike serial manipulators, with PMp-s all types of kinematical pairs are used. However, the most frequently utilized are the mono-mobile kinematical pairs of rotation (R) or translation (T), the bi-mobile cylindrical pairs (C) and tri-mobile spherical pairs (S).

We follow the topological structure of kinematical chains with actuators (KCA), both for the general case of spatial PMp-s (fig. 1a-d) and for the planar PMp-s (fig. 1e).

3.1. KCA type SCS (fig. 1a) is an open kinematical chain KC (1+2) (formed from two elements 1 and 2) which link together two platforms, of which one \(- p_0 \) (with spherical articulation \(A_0\) - can be fixed, while the other is the mobile platform \(p\) (with spherical articulation \(B\)), whose movement is controlled by actuators.

If it is noted with \(M_{sq}\) the mobility of KCA type SCS compatible with axis \((\Delta)\), this is equal to 3, being represented by the active translation motion along the axis (of bar 2 relative to bar 1) and by two passive rotational movements (of bars 1 and 2) about the axis.
The mobility of PMp with \( x \) KCA is calculated with the general formula of the manipulators with \( x-1 \) independent contours of rank 6 [1]:
\[
M = (2 \cdot 1 + 3 \cdot 2) \cdot x - 6 \cdot (x - 1) = 3 \cdot x
\]  
(3.1)
From the equation (3.1) results \( x = 6 \), that leads to PMp with six actuators type SCS (fig. 2), as the Gough-Stewart platform, with six active mobilities (the translations of pistons 2 in cylinders 1) and 12 passive mobilities (the non-controlled rotations of pistons 2 and of cylinders 1 about the axis (\( \Delta \)).
The mobility of PMp (fig. 2) can be immediately verified with formula [1]:
\[
M = \sum_{m=1}^{3} mC_m - \sum_{r=2}^{6} rN_r = (2 \cdot 1 + 3 \cdot 2) \cdot 6 - 6 \cdot 5 = 18
\]  
(3.2)
Indeed, the 18 mobilities are identified from 6 active mobilities (realized by the \( \xi_{31} \) translations) and another 12 passive mobilities, these being the potential \( \varphi_{1,5} \) and \( \varphi_{2,\Delta} \) rotations.
Both the inferior fixed platform and the mobile superior one have, in general, a hexagonal form (fig. 2a) with the tips \( A^0_i \) \( (i = 1 \cdots 6) \) and \( B_i \) \( (i = 1 \cdots 6) \), respectively.
In particular cases, the mobile platform can have a triangular form (fig. 2b) when the spherical articulations \( B_i \) \( (i = 1 \cdots 6) \) coincide or are very close to one another (fig. 2c).

![Fig. 2](image)

Since the solution of locating of the two spherical articulations (fig. 2b) is difficult to realize in practice, the optimal solution is for one of the two spherical articulations \( B_3 \) to connect the cylinder of a actuator (1) to the rod of the other actuator piston (2), as shown in figure 3a (in detail) and in figure 3b in ensemble (similar to Stewart’s platform). If the mobile platform \( p \) is reduced to a bar with two spherical articulations \( B_i \) \( (B_1 = B_2 = B_3) \) and \( B_j \) \( (B_4 = B_5 = B_6) \), respectively (fig. 4), the PMp allows the bar movement \( B_3p \) in any position inside the work space. In this case another passive mobility results, represented by the \( p \) bar rotation about axis (\( \Delta \)) which connects points \( B_1 \) and \( B_4 \).
In the particular case when the mobile platform (\( p \)) is reduced to a point \( B \) is obtained the manipulator type Mianowski [4], with three parallel KC-s type SCS (fig. 5a).
With the help of this manipulator with three active mobilities \( (s_{21}) \) and six passive mobilities (fig. 5b) point \( B \) can be positioned in any position within the work space.

The three independent linear displacements \((s_{21})\) correspond to the three Cartesian coordinates of point \( B \) in the three-dimensional space.

3.2. **KCA type RCS** (fig. 1b) has mobility \( M_s = 2 \), which means an active linear displacement of piston 2 in cylinder 1, as well as a passive rotation of piston 2 about axis \( \Delta \) (fig. 6a). To find the number \( x \) of actuators for a PMp, is applied the formula:

\[
M = (1 \cdot 1 + 2 \cdot 1 + 3 \cdot 1) \cdot x - 6 \cdot (x - 1) = 2x
\]

(3.3)

from which is obtained the solution \( x = 3 \), which corresponds to the kinematical schema from figure 6b. The mobility of this PMp (fig. 6b) is calculated with formula [1]:

\[
M = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r = (1 \cdot 1 + 2 \cdot 1 + 3 \cdot 1) \cdot 3 \cdot 6 - 2 = 6
\]

(3.4)

The six mobilities are achieved by three active translations \((s_{21})\) of pistons 2 about cylinders 1, while the other three mobilities are three passive rotations of pistons 2 about the \( \Delta \) axis.

This axis is a straight line drawn from the spherical joint center in \( B \), having the direction perpendicular to the axis of the rotation joint in \( A_0 \) (fig. 4a).

3.3. **KCA type RSS** (fig. 1c) has two mobilities about axis \((\Delta)\), one active, being obtained as the rotation of bar 1 about the fixed point \( A_0 \), and one passive, represented by the rotation of bar 2 about the straight line \( AB \). The number of KCA-s type RSS (fig. 8a) from the make-up of PMp is deduced with the help of formula

\[
M = (1 \cdot 1 + 3 \cdot 2) \cdot x - 6 \cdot (x - 1) = 2 \cdot x
\]

(3.5)

from which results \( x = 6 \), which corresponds to the kinematical schema in figure 8b.

The mobility of PMp (fig. 8b) is calculated with formula:

\[
M = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r = (1 \cdot 1 + 3 \cdot 2) \cdot 6 - 6 - 5 = 12
\]

(3.6)

From the 12 mobilities of PMp, 6 mobilities are active (represented through the rotations about points \( A_i \) \((i = 1...6)\) and the other six mobilities are passive (being represented through the rotations of bars 2 about the straight line \( AB \), as shown in figure 8a.)
3.4. **KCA type RRS** (fig. 1d) uses only one mobility to maintain point B on axis \(\Delta\) at an instant moment. This mobility \(M_r = 1\) is obtained through the rotation of bar 1 about the fixed point \(A_0\) (fig. 9a). The number of KCA-s type RRS from the make-up of PMP is calculated with the mobility formula

\[ M = (1 \cdot 2 + 3 \cdot 1) \cdot x - 6 \cdot (x - 1) = 1 \cdot x \]  

from which is deduced \(x = 3\).

![Fig. 9](image)

The mobility of this parallel manipulator (fig. 9b) with three PKC-s is verified with the formula [1]:

\[ M = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r = (3.7) \]

\[ (1 \cdot 2 + 3 \cdot 1) \cdot 3 - 6 \cdot 2 = 3 \]  

In this case all mobilities are active, being the rotations of the three bars 1 from each of the three actuators RRS (fig. 9a).

3.5. **KCA type RRR or RTR** (fig. 1e, 10a) have parallel axes for the three articulations; the mobility about axis \(\Delta\) is equal to one \(M_r = 1\).

![Fig. 10](image)

For this case is found the number of actuators in a planar workspace of rank \(r=3\) and, therefore, the synthesis equation it is as follows:

\[ M = (1 \cdot 3) \cdot x - 3(x - 1) = 1 \cdot x \]  

(3.9)

From equation (3.9) is deduced \(x=3\), which represents the number of actuators connecting a mobile plate \(p\) to a fixed one in the plane (fig. 10b). It is identified in this case a triadic KC [3] formed from the triangular plate \(p\) and three binary bars 2.

4. **Topologic Structural Synthesis of PMP-s with Complex KC-s**

The complex KC-s are realized as planar chains with one or two closed contours (fig. 11) or as spatial KC-s with one closed contour (fig. 12).
Beginning from the mono-contour and mono-mobile planar KC (fig. 11a) must be calculated the number of LCA-s which are connected to a mobile platform through kinematical pair type tetra - mobile to form a PMp. To this purpose are written the equations of mobility:

\[ M = (1 \cdot 4 + 4 \cdot 1) \cdot x - [(3 \cdot x + 6(x - 1))] = 1 \cdot x \]  

(4.1)

from which results \( x = 3 \).

We consider the planar KC type mono-contour and bi-mobile (fig. 11b) and maintain the same conditions for the connection to the platform through the tetra - mobile joint. The mobility equations written in this case are as follows:

\[ M = (1 \cdot 5 + 4 \cdot 1) \cdot x - [(3 \cdot x + 6(x - 1))] = 2 \cdot x \]  

(4.2)

from which results \( x = 3 \).

In the hypothesis that the bi-contour and bi-mobile KC (fig. 11c) is used with the same tetra - mobile connection to the platform, the mobility equations are written:

\[ M = (1 \cdot 8 + 4 \cdot 1) \cdot x - [(3 \cdot 2x + 6(x - 1))] = 2 \cdot x \]  

(4.3)

resulting the solution \( x = 3 \).

If is used the spatial KC with planar contour (fig. 12a), the connection of LCA to the mobile platform can be realized through the spherical joint. The mobility equations have the expressions:

\[ M = (1 \cdot 6 + 3 \cdot 1) \cdot x - [(3 \cdot x + 6(x - 1))] = 2 \cdot x \]  

(4.4)

from which is deduced \( x = 3 \). In the situation of a mono-contour spatial KC (fig. 12b), if the connection to the platform is maintained through the spherical joint, the mobility equations are of the form:

\[ M = (1 \cdot 2 + 3 \cdot 4) \cdot x - [6 \cdot x + 6(x - 1)] = 4 \cdot x \]  

(4.5)

From equation (4.5) is obtained \( x = 3 \), what leads to the kinematical schema of PMp from figure 13.

5. Conclusions

Starting from a KCA with module topologic structure, PMp-s can be designed both as spatial kinematical schemas with six or three KCA-s and as planar kinematical schemas with three KCA-s. For this a new mobility formula was applied, which considers the degrees of freedom of the joints used and the rank of the independent closed kinematical contours. New technical solutions are proposed for the transformation of rotation movement into translation movement, so that rotation actuators, such as electric motors, can be used.

References