ON THE EFFECTS OF THE REDUNDANT CONSTRAINTS FROM MECHANISMS

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Abstract: A main aim in mechanical transmissions’ optimization is to remove the structural defects that are due to the hyperstatical constraints. These statically undetermined constraints become active when manufacturing or assembling deviations appear, trying to block up the transmission. Based on a representative example, in this paper it is presented a method of structural modeling for mechanisms with structural defects, and it is proposed a program for testing the effects of deviations on the closed-loop kinematic chain behavior. Then, there are numerically simulated its motions and forces for different deviations from the fundamental model of the mechanism.

1. Introduction

Theoretically, when the mechanism manufacturing is considered ideal, the elastic and thermal deformations are considered null, friction is non-existent and the strength of materials unlimited, the hyperstatical constraints don’t influence the motions and forces transmission. But in real conditions, the mechanical transmission’s deviations, due to its manufacturing and assembling, are unavoidable; at the same time, during the transmission’s functioning, there are possible elastic and thermal deformations of its elements. These hyperstatical constraints (from the transmission theoretical model) that, in real conditions, have a negative influence on the mechanism manufacturing and/or operating are further called as structural defects.

A main problem of mechanical transmissions is the increase of their reliability. A main direction in solving this problem is referring either to the elimination of structural defects (that are due to the statically undetermined constraints) or to the reduction of these constraints’ negative effects (by making a compromise between the optimization costs and their effects on the transmission reliability); in the presence of manufacturing and assembling inaccuracies, these constraints tend to block up the transmission and introduce supplementary stresses, hardly reducing their life-cycle.

An intuitive method for the structural modeling and testing of deviations effects on the transmission functions and mechanisms behavior is proposed in this paper; the proposed method is based on the strictly analytical method for the structural defects elimination, which was formulated in [3,4,5].

![Fig. 1. A 4R mechanism used in grained materials extraction](image)

The modeling is presented on the base of a representative example: a four-bar mechanism from an extractor mechanism used in grained materials extraction from silages – Fig. 1; in real conditions, the
mechanism is characterized by relatively high deviations that can lead to the breakage of worm 11.

The structural and kinematical modeling of a four-bar mechanism (4R) used in an extractor is further presented; the modeling, based on an intuitive method [1, 2], establishes the degree of freedom and the qualitative transmission functions and identifies the redundant constraints. It is also proposed a statically determined variant of mechanism (Fig. 2); in order to simulate the deviations influence, three adjustment joints are inserted into the isostatic variant of mechanism (Fig. 3).

**Tab. 1 The structural analysis of the mechanism**

<table>
<thead>
<tr>
<th>Identification and structural modeling of the kinematical joints</th>
</tr>
</thead>
<tbody>
<tr>
<td>The axes of the four revolute joints are parallel to y; therefore, their structural modeling is identical:</td>
</tr>
<tr>
<td>( f_A = f_B = f_C = f_D = 1 );</td>
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<tr>
<th>Identification of the redundant constraints from the one-loop kinematical chain</th>
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<tr>
<td>a) Because the axes of the joints A, B, C and D are parallel to the z axis, the mechanism is planar and, therefore, by breaking the frame it is obtained an open chain 0-1-2-3-4, which has:</td>
</tr>
<tr>
<td>( (f_{4,0}) = (v_x, \omega_y, v_z) \Rightarrow f_{4,0} = 3 );</td>
</tr>
<tr>
<td>b) Establishing the degree of constraint, inlaid by the open chain, between the extremal elements 4 and 0:</td>
</tr>
<tr>
<td>( c^* = c_{4,0} = 6 - f_{4,0} = 6 - 3 = 3 \Rightarrow 3 ) constraints of equations: ( \omega_x = 0; \omega_z = 0; v_y = 0 );</td>
</tr>
<tr>
<td>These constraints become redundant after 4 and 0 become one piece again (4=0).</td>
</tr>
</tbody>
</table>

**Establishment of the degree of freedom \( M \)**

\( M = 6 \cdot n - [(c_A + c_B + c_C + c_D) - c^*] = 6 \cdot 3 - (20 - 3) = 1 \Rightarrow \) one independent motion

**A qualitative establishment of the transmission functions**

\( L = 2 > M = 1 > 0 = \) the kinematical chain can function as a mechanism.

\( M = 1 \) → an external independent motion: \( (\phi_1, \omega_1 = \frac{d\phi_1}{dt}, \omega_3 = \frac{d\phi_3}{dt}) \);

→ a transmission function for moments: \( T_1 = T_3(\phi_1, T_3) \).

\( L-M = 1 \) → a transmission function for motions: \( \phi_3 = \phi_3(\phi_1), \omega_3 = \omega_3 = \frac{d\phi_3}{dt} \), \( \omega_3 = \frac{d\omega_3}{dt} \);

→ an external independent moment: \( T_3 \).

**The mechanism degree of statical indetermination**

The degree of statical indetermination \( S \) of the one-loop mechanism (the degree of hyperstaticity) is equal to the number of kinematically redundant constraints:

\( S = c^* = 3 \Rightarrow 3 \) statically undetermined constraints (hyperstatical) of equations: \( \omega_x = 0, \omega_z = 0, v_y = 0 \).

These constraints’ elimination can be done by introducing the mobilities \( \omega_x, \omega_z, v_y \).

An example: introduction of three revolute joints in the mechanism (see Fig. 2): joint F introduces a passive mobility among x axis, while joints E and G, with the axes parallel to z, insert the passive motions \( \omega_z \) and \( v_y \), \( \Rightarrow f_{4,0} = 6 \) and, implicitly, \( c^* = c_{4,0} = 0 \) hyperstatical constraints => the mechanism (see Fig. 2) is statically determined.
Further, it is proposed a simulation program for the analysis of the influence of the deviations from the mechanism’s planar configuration, on the dependent angular displacements from all joints (Fig. 2); by means of these displacements it can be modeled the mechanism pre-tensioning moments as products between a modulus of elasticity and the angular displacements from joints E, F, G, which were previously established.

2. Structural and Kinematical Modelling of the 4-bar Mechanism

In given constructive, functional and technological conditions, the elimination of structural defects from mechanisms can be done based on the following algorithm [3,4,5]:

- The structural parameters are established on the mechanism’s theoretical structural model;
- The passive kinematical constraints and the theoretical solutions for eliminating them from the mechanism’s theoretical model are identified;
- In terms of the deviations from the structural model given by the real functioning conditions, the passive kinematical constraints that become structural defects are identified; the rational ways of eliminating them are then selected.

The previous algorithm is exemplified on a one-loop mechanism of 4R type (Fig. 1, b,c), part of an extractor used in silages (fig. 1, a). In theoretical conditions, the axes of joints A, B, C and D are parallel, the clearances and frictions inside the joints are null, the mechanism elements are considered non-deformable and with a perfect technological processing etc. In this case, for establishing the degree of freedom and the structural defects of the considered mechanism, the algorithm from Tab. 1 can be applied.

If the deviations exceed certain limits, the hyperstatical constraints become structural defects, as they tend to overstress the kinematical chain, to block up the mechanism.

The structural defects are eliminated by inserting the mobilities that are indicated by the equations of hyperstatical constraints (see Tab.1).

Then, it is made the kinematical analysis for the isostatical mechanism (7R), using the homogenous operators method: 

\[ T_{10} T_{21} T_{32} T_{02}=I, \]

in which \( T_{xy} \) represents the transformation matrix from the reference frame of element \( x \) to the reference frame of element \( y \), and \( I \) is the unit matrix.

By solving the system of equations obtained from the previous matrix, it becomes possible to model the influence of the geometrical deviations on the relative motions from the joints of the isostatical mechanism 7R.

The analytical expressions of the joints relative motions are used directly in the simulation programs.

3. Numerical simulation

In order to control the errors, three distinct joints (\( \alpha, \beta \) and \( \Delta y \) - Fig. 3) are inserted in the considered mechanism (\( \alpha \) allows a rotation around \( x \), \( \beta \) - around \( z \) and \( \Delta y \) – a translation along \( y \)): the joints allow the introduction of angular / linear displacements that model the errors from the planar configuration (after moving with one angular / linear step in each of the joints used for adjustment, the respective joint is considered frozen).

The program is run with the following numerical values for the dimensions of the mechanism elements:

- \( L_{xy}=1.246 \) m, \( AB=0.12 \) m, \( CD=1.18 \) m, \( BC=0.4 \) m and \( \phi_1=[0,2\pi] \) rad.

- The deviations are introduced in the mechanism, through the adjustment joints \( \alpha, \beta \) and \( \Delta y \), with the following range of variation: \( \alpha, \beta \in [-3^\circ, +3^\circ] \), \( \Delta y \in [-0.12 \text{m}, +0.12 \text{m}] \). Based on these numerical data, a program for the kinematical modeling of the mechanism in the spatial state (with non-zero values for \( \alpha, \beta \) and \( \Delta y \) deviations) was then elaborated. Firstly, the influence of each deviation on the angular displacements was numerically analyzed; afterwards, the influence of the combinations of different deviations on the same entities was numerically analyzed. The representative variations of the angular displacements are represented in Fig. 4 and 5.

According to Fig. 4, it can be observed that the combined influence of the \( \alpha, \beta \) and \( \Delta y \) deviations (considered with the extreme values) on the displacements from joints B, C and D can be characterized as follows: the deviations don’t practically influence the relative motions from joint B (for this reason, this diagram is not shown); the combined influence of deviations on the displacements from joints C and D is very small, even if the deviations are relatively big.

In Fig. 5, a, b and c there are represented the variations of the angular displacements from the theoretically passive joints E, F and G, for deviations values equal to: \( \alpha = -3^\circ,...,+3^\circ \), \( \beta = \Delta y = 0 \). It can be observed that deviation \( \alpha \) and the extreme angular displacements from joints E, F, G are increasing together; the maximum values appear in joint E and the minimum ones in joint G.
In Fig. 5, d and e there are represented the variations of the angular displacements from the passive joints E and F, for deviations values equal to: $\beta = -3^\circ, +3^\circ$, $\alpha = \Delta y = 0$. It can be observed that deviation $\beta$ has comparable influences on the displacements from joints E and G, but are backwards; the influence on the joint F displacement is very small (approximately 3 times smaller).

In Fig. 5, f it is represented the variations of the angular displacement from the passive joint E, for deviations values equal to: $\Delta y = -0.12, +0.12$ m, $\alpha = \beta = 0$. Nearby, in Fig. 5, g, h and i there are illustrated the variations of the displacements from for joints E, F, G the sets of values $(\alpha = \beta)/\Delta y = -3^\circ/-0.12$, $-2^\circ/-0.08$, $-1^\circ/-0.04$, $0/0$, $+1^\circ/+0.04$, $+2^\circ/+0.08$, $+3^\circ/+0.12$.

(a) Angle $\phi_{i\_C}$
(b) Angle $\phi_{i\_D}$

Fig. 4. The influence of deviations on the dependent motions
It can be observed that deviation $\Delta y$ has no influences on the displacements from joint F, but the influences on the displacements from joints E, G are equal and of opposite direction. According to Fig. 5g, h and i, the influence of deviations $\alpha$ and $\beta$ diminishes the influence of deviation $\Delta y$ in joints E and G with approximately a half; instead, it introduces angular displacements in the middle joint F, equal approximately to a half of the other two joints displacements.
4. Conclusions

In order to identify the effects of the deviations on the transmission functions and on the closed kinematical chain behavior, a program for testing the influence of the deviations from the mechanism planar configuration is proposed in the paper.

A. The program is built using Maple software, and is intended to establish:
   a. the influence of deviations from the planar configuration, on the angular displacements from joints B, C and D;
   b. the influence of the deviations from the planar configuration, on the angular displacements from joints E, F and G; when structural deviations appear, the passive motions from joints E, F and G become active and the 4R mechanism becomes a spatial mechanism of 7R type;
   c. the deviations’ influence on the pre-tensioning of the hyperstatical mechanism (S=3), through the angular displacements from joints E, F and G, which are considered as replaced by twistable elastic elements; therefore, the pre-tensioning moments can be modeled as products between a modulus of elasticity and the angular displacements of joints E, F, G, which were previously established.

B. In order to control the deviations, three distinct joints $\alpha$, $\beta$ and $\Delta y$ (Fig. 3) are inserted in the considered mechanism ($\alpha$ allows a rotation around x, $\beta$ - around z and $\Delta y$ – a translation along y); the joints allow the introduction of angular / linear displacements that model the deviations from the planar configuration (after moving with one angular / linear step in each of the joints used for adjustment, the respective joint is considered frozen).

C. Different values, separately and combined, are given to the displacements from the adjustment joints and their effects on the parameters established at point A are numerically analyzed. Afterwards, by processing the results which are obtained by simulation, there are formulated conclusions regarding the structural optimization under known constructive, technological and functioning conditions.

The angular displacements from the theoretically passive joints E, F and G grow together with the increase of the deviations from the planar configuration: $\alpha$, $\beta$ and $\Delta y$. Under the premise that these joints are inexistent, pre-tensioning moments appear, moments that are equal to the product between the angular displacements and the elements modulus of elasticity. Obviously, due to the reduced elements elasticity, pre-tensioning moments of big values can appear in the closed chain; in these conditions, the mechanism durability is compromised, being necessary to insert supplementary adequate mobilities.

The modeling and numerical simulation highlight the necessity of an attentive numerical study about the influence of structural defects ($\alpha$, $\beta$ and $\Delta y$) in the design stage and, based on the obtained results, to identify the theoretically passive mobilities that are absolutely necessary and the optimal dimensioning of tolerances and adjustments.

References